Solving Constrained Path, Moving Target Search Problems Using Multiple Searchers

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Abstract — **All previous research involving search for a moving target focuses on a single searcher. We develop an optimal branch and bound procedure and six heuristics for moving target constrained path problems with multiple searchers. Our optimal procedure outperforms existing approaches for single searcher problems. For more than one searcher, the time needed to guarantee an optimal solution is prohibitive. Our heuristics represent a wide variety of approaches and consist of one based on solving partial problems optimally, two based on the expected number of detections, two genetic algorithm implementations, and local search. A heuristic based on the expected number of detections obtains solutions within two percent of the best known for each one, two, and three searcher test problem considered. For one and two searcher problems, the same heuristic's solution time is less than that of other heuristics. A genetic algorithm implementation performs acceptably for one and two searcher problems and highlights its ability, effectively solving three searcher problems in as little as 20% of other heuristic run-times.**

Key Words — Search for Moving Objects, Detection, Multiple Searchers, Constrained Path Planning, Team Effort, Network Optimization, Branch and Bound, Shortest Path, Combinatorial Optimization, Heuristic Methods, Moving Horizon, Artificial Intelligence, Genetic Algorithm, Local Search, Stochastic Processes.

SOLVING CONSTRAINED PATH, MOVING TARGET SEARCH PROBLEMS USING MULTIPLE SEARCHERS

We extend the single searcher model proposed by Eagle and Yee (1990) to multiple searchers. Both single and multiple searcher models use discrete time with a single target's motion modeled as a Discrete Time Markov Chain. The target is constrained to a single cell within a grid each time period, and has its movement alternatives, between time steps, restricted to adjacent cells. The initial probability distribution for the target and the target's Markovian transition matrix are known.

The initial position(s) for the searcher(s) has to be specified. The searcher(s) has the same type of movement restriction as the target and a limited time to search. The search path's effectiveness is the cumulative probability of detection along the searcher's path(s) where detection occurs with a specified probability when the searcher and target occupy the same cell. Our implementation allows more than one searcher to searcher to search the same location at the same time. The random search law is used, which allows the detection rates of each searcher to be added. Each time period, the probability distribution for the target throughout the area is Bayesian updated for no-detection.

An appropriate formulation for the multiple searcher problem, an extension of the single searcher problem of Eagle and Yee (1990), follows the introduction of notation.

- Indices:
  - * i, i',k cell,
  - * j searcher,
  - * t time step (t = 1,2,...,T),
  - * ω path (where $\omega(t)$ is the cell occupied at time t).
- Data:
  - * α_{ij} detection rate for searcher j in cell i. The probability of detection in a given cell is 1 - exp($-\alpha_{ij}$),
  - * Ω set of all feasible target paths,
  - * C_i set of cells adjacent to cell i,
  - * p_ω probability of target following path ω,
  - * s_j starting cell for searcher j at time zero.
- Binary Variables:
  - * $x_{i,\,\omega(t),j,t} = 1$ if searcher j moves from cell i at time t-1 to cell $\omega(t)$ at time t and 0 otherwise.
- Formulation:

Maximize

$$PD = (1 - \sum_{\omega \varepsilon \Omega} p_\omega \exp(-\sum_t \sum_j \alpha_{\omega(t)j} \sum_{i|\omega(t)\varepsilon Ci} x_{i,\,\omega(t),\,j,\,t}))$$

subject to the constraints:

$$\sum_{i'\varepsilon C s_j} x_{s_j,\,i',\,j,1} = 1 \ \forall j \qquad (1)$$

$$\sum_i \sum_{i'} x_{i,i',j,t} \le 1 \quad \forall\, j,t \qquad (2)$$

$$\sum_{i|i'\varepsilon C_i} x_{i,i',j,t-1} = \sum_{k\varepsilon C_{i'}} x_{i',k,j,t} \quad \forall i',\ t > 1, j \qquad (3)$$

The formulation maximizes the probability of detection (PD), within the set of feasible paths Ω, subject to the constraints:

(1) Each searcher's initial search effort (t=1) must be in a cell adjacent to the starting position;

(2) Each searcher can move at most once between time periods. Since the maximum objective function value is sought, the exclusion of this constraint could result in multiple paths for each searcher;

(3) All search effort has to be done within the set of adjacent cells, at any time step, for any given searcher.

Trummel and Weisinger (1986) show the path constrained search problem for a stationary target is NP-Complete. An example highlights the problem's complexity. A single searcher using five time steps to search a nine cell problem has approximately 1,000 feasible paths to choose from. The same problem with 10 time steps has about 1,000,000 feasible paths. This problem with three searchers has about 1.0 E18 feasible paths. The path constrained search problem for a moving target with multiple searchers is at least as hard, and by being so, the main thrust of this paper is the development, testing, and evaluation of relatively fast and robust heuristics. The heuristics considered are: one based on solving partial problems optimally, two based on the expected number of detections, a genetic algorithm, a hybrid genetic algorithm that incorporates other heuristics, and local search. This paper also develops an optimal branch and bound procedure which outperforms other optimal procedures reported in the literature for one searcher problems.

SEARCH PROBLEMS AND MOTIVATION

The Operations Research literature contains numerous books and published articles on stationary target problems. The consensus of the research community is that the framework for these problems was laid down by the 1942 United States Navy Antisubmarine Warfare Research Group in response to the Atlantic German submarine threat (Koopman (1946)). Subsequent work by many researchers took the stationary target problems into a mature state where solutions are available for the most common problems and improvements are hard to find (Stone(1975)).

The case for a lone searcher of a lone searcher looking for a single moving target has also been widely studied and can be divided into two major classes: Two-sided search and one-sided search.

Two-sided search problems consider situations where the target is aware that a search effort is being carried out against him and attempts to avoid detection or capture. Game theory is the natural tool here (see Thomas and Washburn (1991), and Eagle and Washburn (1991)). One-sided search problems assume either the target is not aware of the search or the target needs to accomplish its own task and it is not willing to evade the searcher. Through this reasoning idea of a Bayesian probability distribution and update of the target's position is straightforward. The one-sided search problems are usually further divided as Optimal Density or Optimal path problems. Both groups in more recent work have dealt with the target motion being modeled as a Discrete Time Markov Chain and the "continuous search" in each time step being modeled by an exponential law of detection.

Optimal density problems tend to be easier problems than optimal path problems since integrality or adjacent movement constraints can be dropped. These problems are well suited to situations when the searcher and target speeds differ by more than an order of magnitude. Brown (1980) made important progress in optimal density problems by developing an algorithm that solves the moving target problem as a sequence of stationary target problems. Washburn (1980) gave a counterpart algorithm for the discrete search effort case as did Stone et. Al. (1978).

Optimal path problems with the characteristics described above are tackled by Stewart (1979) and (1980) using an optimal branch and bound procedure. Eagle and Yee's (1990) branch and bound procedure obtains bounds by using the Frank-Wolfe algorithm to solve a subproblem where integrality restrictions are relaxed.

Another interesting model for Optimal Path Problems is the continuous time and space case where the constraints on the searcher's motion are given by a set of differential equations that the searcher's path has to obey. Oshumi (1991) is a good example of such a model.

According to the survey by Weisinger, Monticino and Benkoski (1991), 125 references are available for one-sided search problems and 61 to search games but *none* are listed for multiple searchers or team effort under the same modeling assumptions. This paper considers moving target constrained path problems with multiple searchers.

ALGORITHMS FOR MOVING TARGET CONSTRAINED PATH PROBLEMS WITH MULTIPLE SEARCHERS

We develop seven algorithms to determine the path(s) that maximizes the probability of detecting a randomly moving target using multiple searchers.

1. Optimal Branch and Bound Procedure (BB)

Two optimal branch and bound procedures exist in the literature for single searcher moving target constrained path problems. Stewart (1979) and (1980) relaxes the searcher's path constraints to obtain a lower bound. Eagle and Yee (1990) maintain the path constraints, but relax the binary condition of the searcher's position. Our approach bounds PD above by calculating the expected number of detections (ED) and thus maintains both the searchers' paths and the binary constraints.

Motivation for using the path or partial path corresponding to the maximum ED is its equivalence to finding the longest path through an acyclic network. Our implementation is adapted from Cormen, Leiserson and Rivest's (1992) Directed Acyclic Graph Longest Path Algorithm which is $\Theta(V+E)$ where V is the number of vertices and E is the number of Edges.

2. Local Search (LS)

This work includes local search (see Papadimitriou and Steiglitz (1982) as a benchmark of how a simple heuristic performs on our test problems.

3. Expected Detection Heuristic 1 (H1)

This heuristic obtains its motivation from the relative ease of calculating the path maximizing ED.

4. Expected Detection Heuristic 2 (H2)

This heuristics expands on H1 by basing the next node added to the searcher's path according to a balance between ED and PD for every feasible move on each time step.

5. Genetic Algorithm (GA)

Genetic Algorithms (see Goldberg (1989) and Holland (1975)) are self improving algorithms that work by means of natural selection, or survival of the fittest.

A characteristic of Genetic Algorithms is the need to set run-time parameters, such as the population size, the probability of cross-over, the probability of mutation, and the number of generations. We automate this process in our GA implementation to provide values that are empirically robust across a variety of problems. This, of course, may limit the efficiency of the algorithm for particular cases but allows the algorithm's results to be more easily generalized to new problem instances.

6. Hybrid Genetic Algorithm (HGA)

The HGA algorithm is the GA which includes in the starting population the three heuristic solutions produced by H1, H2 and the path that maximizes ED.

7. Moving Horizon (MH)

The MH heuristic breaks the true problem into subproblems which are optimally solvable within a reasonable amount of computer time.

PRESENTATION MOTIVATION

Even though the main thrust for development of efficient and effective algorithms to deal with search problems comes from the military community, many civilian applications can also benefit, such as: search for lost hikers, endangered animal species and shoals of fish for commercial reasons.

During the conference, we plan to present computational results of the algorithms developed as well as discuss in more detail their characteristics.

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