Direction of Arrival and Synthesis Problem through Beam Forming using Parsimonious Representation

Sébastien Rondineau(1), Benjamin Fuchs(2), Jean-Jacques Fuchs(3)

(1) Solentech – Solution, Engineering & Technology
Porto Alegre, RS – Brazil
s.rondineau@solentech.com.br

(2) IETR/CNRS
Université de Rennes 1, Rennes
France

(3) IRISA/CNRS
Université de Rennes 1, Rennes
France

Abstract—**In both reception, such as direction finding, and transmission, such as synthesis far field patterns, parsimonious representation of beam former has numerous advantages that compensates its heavier computing load. On one hand, in transmission, the synthesis problem is an optimization that guaranties to find not only the best pattern, but also minimizes the number of necessary sources that achieves the desired far field pattern. On the over hand, with the reception problem, the parsimonious representation of beam formers results in an algorithm independent of the array geometry, that can handle both coherent and incoherent signals, and doesn't rely on any prior assumption on spatial stationarity. It uses global matching filtering in a way that its performances are close to the Cramer-Rao bounds, and most of the time better than MUSIC.**

Keywords; beam forming; direction of arrival, MUSIC, sparse matrix, goniometry, optimization, global matching filter.

I. Introduction

Due to its many applications in electronic warfare communications and signal processing, array pattern synthesis has been extensively investigated over the last several decades [1]-[7]. It consists of determining the array excitation that leads to a specified radiation pattern.

The synthesis of narrow beam and low sidelobes is a frequently encountered problem that has first been solved by Dolph [8] for uniformly spaced linear arrays. He obtained a closed form that optimizes the compromise between beamwidth and sidelobe level.

Since then, numerous papers have been proposed to generalize and extend Dolph's work to deal with any sidelobe envelope and arbitrary arrays. An arbitrary array can have a non linear shape and may be composed of non uniformly spaced or non isotropic or even non identical elements. Analytical solutions are of course not available for such problems.

Many numerical synthesis methods have thus been proposed. Drawing up an exhaustive list would be impossible but one may distinguish two main classes. Many methods are indeed iterative and based on weighted least square algorithms [9]–[12]. A good review can be found in [13]. Another important class of pattern synthesis methods are inspired on adaptive array algorithms [14]–[18] that adjust their patterns so as to maximize the signal to noise ratio while rejecting a set of prespecified interferers.

In both classes, an ad hoc iterative scheme is implemented that, at each step, computes the difference between the desired pattern and the currently obtained pattern and adjust the weights in the least square criterion or the angles and powers of the interferers in the adaptive array schemes before computing the new "optimal" pattern. Moreover, quite often in these schemes the phase of the optimal pattern needs to be defined and clever phase adaptation means are designed to avoid a wrong choice of phase that precludes optimal performance.

This recursive feedback scheme is in general fully automatized and requires no outside intervention. This procedure is iterated until a suitable pattern is obtained. According to the authors convergence generally occurs after a few steps.

As rightly pointed out in [19], [20], if the synthesis problem is not convex, there is no guarantee that the absolute optimum is reached and indeed the problems, as they are considered in most of these papers, are not convex.

Moreover while for regular arrays with isotropic sensors, optimal methods, such as Dolph's technique tell us what the achievable performance is, for arbitrary arrays no such guidelines pre-exist. It is then difficult to both set a priori reasonable constraints and draw a definite conclusion, regarding the achievable performance of these arrays using these algorithms.

Except for [19], [20], where interior point methods are used to constraint the beam pattern level of linear, adaptive and broadband arrays, very few papers have investigated the potentialities of convex optimization in antenna pattern synthesis problems.

In the case of direction of arrival problems, correlation between waveforms generally appears in the case of multipath propagation and can severely degrade the performance of an antenna array system. Most source localization techniques, including those that are eigen structure-based such as MUSIC [21, 22], encounter difficulties only when the signals are perfectly correlated or coherent. But in practice, however, significant degradations arise when the signals are highly correlated.

Though of high practical importance, the highly correlated or the fully correlated (coherent) case did not receive the attention it deserves [23] and in general only some preprocessing schemes whose aim is to deccorelate the signals are proposed to improve the performance or reduce the degradations of techniques that are build and developed for uncorrelated sources. A preprocessing technique known as spatial smoothing has been proposed in [24] and further investigated in [25, 26]. The main drawback of this approach, besides being only applicable to regular array geometries, is the reduction of the effective array aperture and hence lower resolution and accuracy. Another technique known as redundancy averaging [27, 28] is known to induce bias in the bearing estimates.

In the sequel we consider a technique that works for any geometry and that is able to handle correlated as well as coherent waveforms. It simply relies on no prior assumption.

In this paper, we present first the parsimonious representation of beam formers, then its application to pattern synthesis and to direction of arrival analysis.

II. Parsimonious Representation

The parsimonious representations are especially used to find a parsimonious representation of a signal (mono, bi- or three-dimensional, 1D, 2D or 3D) aiming a compression of information, for example. The idea is simple, instead of representing in a strict sense the signal in a base (orthogonal or not) and if required canceling the small coefficients to compress information, one proposes to represent it in a redundant base containing a number of vectors (components, atoms) much higher than the dimension of the signal in which the signal will have an infinite number of representations which one tries to find most parsimonious, the one using the least possible components. The more redundant the base is and the more parsimonious the parsimonious representation will be. It remains then to develop the way to build these bases that fit the signals and the algorithms which make it possible to find with the lowest costs, a parsimonious representation, in the absence of *The* parsimonious representation.

One can also use this type of approach when the signal to be represented has an exact parsimonious representation in a redundant base which is then, in general, well characterized. It is the case of the global matching filters. The goal is then precisely to find this exact parsimonious representation.

In the context of the goniometry which interests us here, one can typically apply this idea by using as observations not the initial data, the snapshots which are in general of null averages but a set of beam outputs pointed in a set of directions uniformly distributed in azimuth and in site. It is necessary that these observations (rebuilt) contain all the information already contained in the initial data (sufficient statistics) and leads to a well conditioned problem. The redundant base should then be created whose each component represents the contribution to this set of beams, a source or a path resulting from a source. Ideally, without disturbances, errors of modeling, errors of discretization, the vector of the observations is then the weighted sum of a small number of components of the base. In reality, it is necessary to take into account these errors and disturbances. One can add to the former base some contributions of the sources, one or more bases modeling the contributions of the disturbances (the noise in the observations etc.) and tolerate an error in the representation. One can, in addition, introduce statistical properties of the observations and thus hope to reach performances close to those of the maximum of likelihood.

It is seen however that the number of parameters to be discretized can increase quickly (for example, if one wanted to model the correlation between the paths resulting from a same source) producing an unacceptable increase of the components to be considered in the base. Such an increase not only penalizes computing time but also the condition number of the problem and, in a certain way, the probability that the representation needed is most possible parsimonious.

In general, one replaces the research of representation the most parsimonious, which could be done only using an exhaustive exploration (and which thus is completely excluded) by the minimization of the standard ℓ_1 norm of the weights and, of course, nothing makes it possible to guarantee that neither the representation thus obtained, nor the most parsimonious representation, is the exact representation, desired. One can notice that, in practice, with simulations, that is generally the case and that the limits of the approach seems close to one of the maximum of likelihood approaches.

The parsimonious representation resolution of the problem can be summarized as follows:

$$\min_x \frac{1}{2}\|Ax-b\|^2+h\|x\|_1,\ h>0 \tag{1}$$

where A contains the description of the pattern of each antenna, b is the desired or measured overall pattern containing errors, x the weight of the sources and h a parameter that regulates the order of magnitude of the errors of rebuilding or the constraints. The representation is independent of the array geometry.

III. Application To Pattern Synthesis

The pattern synthesis using parsimonious representation can be used in any application that needs to transmit power and performs electronic scanning. The maximum speed of the scan is directly linked to the studied bandwidth and the resolution of eq. (1). To ensure that the resolution of eq. (1) leads to *The* absolute optimal solution, the filling of A, b and h is done in such a way that we deal with a convex optimization problem.

An interesting property of that technique is that the solution obtained has always the minimum number of necessary sources.

Notice that each antenna can have its own pattern description, that the patterns can be either theoretical or measured. Coupling between antenna element is also included via the impedance matrix associated to the array.

To illustrate the potentialities of this technique, figure 1 shows the far field pattern of the synthesis of a non uniform sidelobe envelop with a uniform linear array (20 isotropic half-a-wavelength-spaced elements).

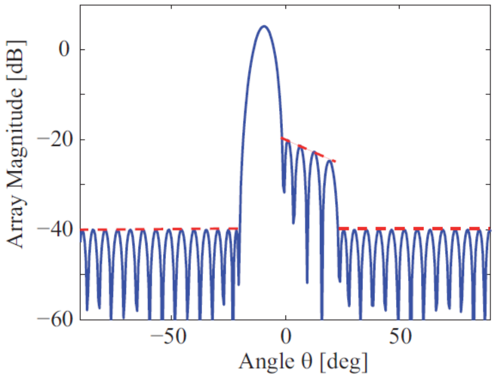


Figure 1. Far field pattern of the synthesis of a non uniform sidelobe envelop with a uniform linear array (20 isotropic half-a-wavelength-spaced elements).

IV. Application To Direction Of Arrival

In the analysis of the direction of arrival, the parsimonious representation of the problem is used to apply Global Matching Filters (GMF) without any assumption on spatial stationarity. Three version of the GMF are used. The brute force one, called GMF-1, where (1) is directly solved. The version GMF-2 where all the information on the antenna, the scene and the noise is passed to the matrix A. In this case A is whitened and normalized and b is whitened. And the approximation of the maximum of likelihood, GMF-3. A is built in such a way that the algorithms to reach the ultimate resolution limit that is a function of the Cramer-Rao bounds [29].

In order to illustrate some of the most significant advantages of the GMF over the MUSIC algorithm, we now consider a linear array of 10 omni-directional receiving antennas, equally spatially distributed at half a free space wavelength. The signal is composed of sequence of QPSK independent symbols. Each algorithm has a grid of 120 outputs.

A. *Uncorrelated sources without calibration error*

In the ideal case of no model error, Table 1 shows that MUSIC's resolution is never as good as the ones of the GMF based algorithms at any signal-to-noise ratio (SNR). On the top of that, the GMF algorithms not only estimate pretty well the directions of arrival of each emitting source, but also predict the associated amplitude levels as well.

Table I. Angular, $\Delta\theta$, and amplitude, ΔA, separations of algorithms of two emitting sources at a level A_0, which guaranty an angular estimation error better than 1° of each emitting source.

SNR	MUSIC	GMF-1		GMF-2		GMF-3	
	$\Delta\theta$	$\Delta A/A_0$	$\Delta\theta$	$\Delta A/A_0$	$\Delta\theta$	$\Delta A/A_0$	$\Delta\theta$
10dB	3°	100%	< 2°	45%	< 2°	23%	< 2°
0dB	6°	18%	< 3°	20%	< 3°	15%	< 3°
-10dB	10°	15%	8°	12%	< 6°	12%	8°

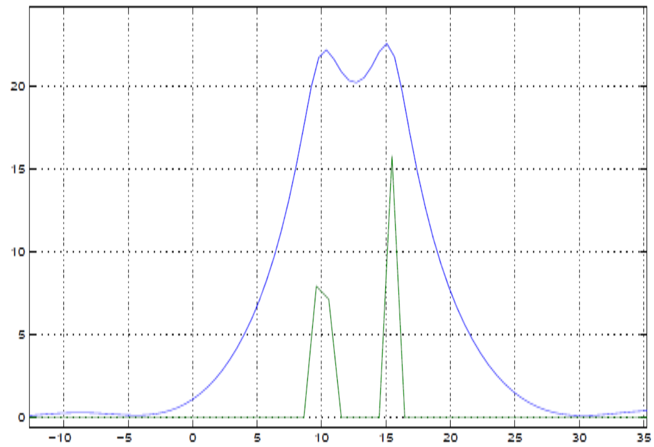
10000 simulations, 150 snapshots and decorrelated sources

In the same manner, it is shown, on figure (2), that MUSIC has difficulties to separate close sources whereas GMF are well suitable, but at the cost of a heavier computing load. More generally, GMF's performances are close to the Cramer-Rao bounds, and most of the time better than MUSIC.

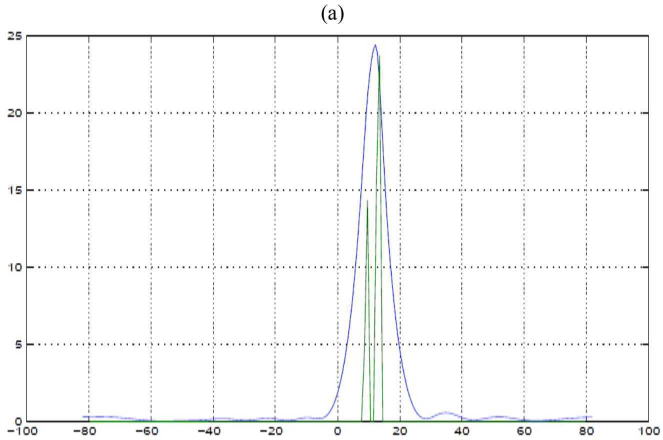
B. *With calibration error*

One takes the former simulations by introducing error on the model. More precisely, one adds, to the nominal vector direction, a noise, composed of a circular complex Gaussian vector, whose covariance matrix is real and one then renormalizes the vector thus obtained with its nominal norm value.

For a high standard deviation of 0.24 associated to the noise, MUSIC doesn't even predict correctly the directions of arrival at the high SNR=10dB. Predictions given by GMF-1 are also incorrect in both direction and amplitude. But the GMF-2 and GMF-3 based algorithms continue to estimate pretty well both angular position and amplitude of the emitting sources, showing their robustness.



(a)



(b)

Figure 2. Outputs of the algorithms MUSIC (blue) and GMF-3 (green) normalized to the same noise level of two emitting sources at the level 0dB, placed at 10 degrees and 16 degrees (a), and 10 degrees and 14 degrees (b). 150 snapshots are considered. The horizontal axis represents the angular position, in degree, on the horizontal plane, and the vertical one shows the power of the output of the algorithms.

C. *Correlated sources*

Table 2 shows that GMF-3 can still detect the correct directions in the presence of any level of correlation, Where as MUSIC tends to detect one source located at the middle of the two real source when the correlation increases. In the case of coherent paths, as shown on Table 2, methods like MUSIC are not appropriate anymore because the row of the matrix of covariance of the snapshots is not equal any more to the numbers of present paths. Considering that the method is based on the contained information in the paths, if traditional techniques succeed in detecting and separating the sources or paths, it is the same for GMF methods with an improved resolution.

TABLE II. ALGORITHM PERFORMANCE COMPARISON WITH CORRELATED EMITTING SOURCES (AT 10° AND 16°)

	MUSIC		GMF-3	
correlation	θ_{10} [°]	θ_{16} [°]	θ_{10} [°]	θ_{16} [°]
0	10.2	15.8	10.2	15.8
0.25	10.2	15.8	10.0	16.0
0.50	9.7	16.1	9.8	16.2
0.75	One source around 13°		10.4	15.6
1	One source around 13°		10.4	15.6

V. CONCLUSION

This paper presented the potentialities of parsimonious representation in pattern synthesis and direction of arrival estimation. This technique opens new promising applications not only in electronic warfare with radar, direction finders, but also in telecommunications in general with the intelligent radio concept IEEE 802.22 for example.

In the transmission case, this representation, that produces a convex optimization problem, ensures to obtain of *The* absolute optimum minimizing the number of necessary feed.

In the reception case, the parsimonious representation of beam formers results in an algorithm independent of the array geometry, that can handle coherent signals, and doesn't rely on any prior assumption on spatial stationarity. It uses global matching filtering in a way that its performances are close to the Cramer-Rao bounds, and most of the time better than MUSIC, and gives not only the angular position of the source, as MUSIC does, but also estimates its signal level.

REFERENCES

[1] A. Schell and A. Ishimaru, ”Antenna pattern synthesis,” in ”Antenna theory,” Part 1, R.E. Collin and F.J. Zucker, McGraw-Hill, New York, NY, 1969, chapter 10.

[2] W.L. Stutzman and G.A. Thiele, ”Antenna theory and design,” Wiley, New York, NY, 1981, chapter 10.

[3] R.C. Hansen, ”Phased Array Antennas,” Wiley, New York, NY, 1998, chapters 3 and 4.

[4] R.S. Elliott, ”Antenna theory and design,” rev. ed., IEEE Press, Piscataway, NJ, 2003, chapters 5 and 6.

[5] R.J. Mailloux, ”Phased array antenna handbook,” 2nd ed., Artech House, New York, NY, 2005, chapter 3.

[6] L. Josefsson and P. Persson, ”Conformal array antenna theory and design,” Wiley, Hoboken, NJ, 2006, chapter 10.

[7] W.L. Stutzman and S. Licul, ”Synthesis method for antennas,” in ”Modern Antenna Handbook,” C.A. Balanis, Wiley, New York, NY, 2008, chapter 13.

[8] C.L. Dolph, ”A current distribution for broadside arrays which optimizes the relationship between beam width and sidelobe level,” Proc. IRE, vol. 34, pp.335-348, June 1946.

[9] R.F.E Guy, ”General radiation-pattern synthesis technique for array antennas of arbitrary configuration and element type,” IEE Proc., vol. 135, Pt. H, no. 4, Aug. 1988.

[10] C.-Y. Tseng and L. J. Griffiths, ”A simple algorithm to achieve desired patterns for arbitrary arrays,” IEEE Trans. Signal Processing, vol. 40, no. 11, pp. 2737-2746, Nov. 1992.

[11] A. Cantoni, X.G. Lin and K.L. Teo, ”A new approach to the optimization of robust antenna array processors,” IEEE Trans. Antennas Propag., vol. 41, no. 4, pp. 403-411, April 1993.

[12] B.P. Ng, M.H. Er and C. Kot, ”A flexible array synthesis method using quadratic programming,” IEEE Trans. Antennas Propag., vol. 41, no. 11, pp. 1541-1551, Nov. 1993.

[13] O.M. Bucci, G. D'Elia, G. Mazzarella and G. Panariello, ”Antenna pattern synthesis: a general approach,” Proc. IEEE, vol. 82, no. 3, March 1994.

[14] J.C. Sureau and K.J. Keeping, ”Sidelobe control in cylindrical arrays,” IEEE Trans. Antennas Propagat., vol. 30, no. 5, pp. 1027-1031, Sept. 1982.

[15] E.C. Dufort, ”Pattern synthesis based on adaptive array theory,” IEEE Trans. Antennas Propag., vol. 37, no. 8, pp. 1011-1018, Aug. 1989.

[16] C.A. Olen and R.T. Compton, ”A numerical pattern synthesis algorithm for arrays,” IEEE Trans. Antennas Propag., vol. 38, no. 10, pp. 1666-1676, Oct. 1990.

[17] P.Y. Zhou, M.A. Ingram and P.D. Anderson, ”Synthesis of minimax sidelobes for arbitrary arrays,” IEEE Trans. Antennas Propag., vol. 46, no. 11, pp. 1759-1760, Nov. 1998.

[18] P.Y. Zhou and M.A. Ingram, ”Pattern synthesis for arbitrary arrays using an adaptative array method,” IEEE Trans. Antennas Propag., vol. 47, no. 5, pp. 862-869, May 1999.

[19] H. Lebret, ”Synthèse de diagrammes de réseaux d'antennes par optimisation convexe,” PhD thesis, University of Rennes 1, France, Nov. 1994.

[20] H. Lebret and S. Boyd, ”Antenna pattern synthesis via convex optimization,” IEEE Trans. Signal Processing, vol. 45, no. 3, pp. 526-531, March 1997.

[21] G. Bienvenu and L. Kopp, “Source power estimation method associated with high resolution bearing estimation” Proceedings of the IEEE ICASSP Conference, Atlanta, p. 153-156, 1981.

[22] R.O. Schmidt. “Multiple emitter location and signal parameter estimation,” Proceedings of the RADC Spectral Est. Workshop, p. 243-258, 1979.

[23] D. Rahamim, J. Tabrikian and R. Shavit, “Source localization using vector sensor array in a multipath environment,” IEEE Trans. on S.P., 52, 11, 33096-3103, 2004.

[24] J.E. Evans, J.R. Johnson and D.F. Sun, “High resolution angular spectrum estimation techniques,” Proceedings of the 1st ASSP Workshop Spectral Estimation, Hamilton, ON, p. 134-139, 1981.

[25] J.S. Thompson, P.M. Grant and B. Mulgrew, “Performance of spatial smoothing algorithms for correlated sources ,” IEEE Trans. on Signal Processing, 44, p. 1040-1046, Apr. 1996.

[26] J. Li, “Improved angular resolution for spatial smoothing techniques,” IEEE Trans. on Signal Processing, 40, p. 3078-3081, Dec. 1992.

[27] D.A. Linebarger and D.H. Johnson, ”The effect of spatial averaging on spatial correlation matrices in the presence of coherent signals ” IEEE Trans. on ASSP, 38, p. 880-884, May 1990.

[28] L.C. Godara, “Beamforming in the presence of correlated arrivals using structured correlation matrix” IEEE Trans. ASSP, 38, p. 1-15, Jan. 1990.

[29] J.J. Fuchs, “On the Application of the Global Matched Filter to DOA Estimation with Uniform Circular Arrays,” IEEE-T-SP, vol. 49, 4, pp. 702-709 , 2001