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LPI Radar Detection: SNR Performances for a Dual Channel Cross-Correlation based ESM Receiver

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***Abstract*— Detection of Low Probability of Intercept (LPI) Radar signal is an important issue for an Electronic Support Measure (ESM) Receiver. The ESM Receiver does not have any knowledge of the modulating code and carrier frequency of the LPI Radar signal, it can only make general assumptions about the occupied bandwidth. The lack of knowledge of the modulating code strongly constrains the maximum obtainable Signal to Noise Ratio (SNR) by a typical ESM Receiver. This paper presents a Receiver architecture which allows an improvement of the achievable SNR: the proposed architecture is based on a dual channel Receiver and combines an Uniform Filter Bank with the Cross-Correlation technique. Theoretical SNR performances, derived for a generic case, will be compared with simulations carried out with realistic phase modulated LPI signals. Measurements done on new generation Elettronica SpA (ELT) Digital Receiver based ESM shall be finally compared with both theoretical and simulation results.**

I. Introduction

ESM Receivers have the main task to detect, measure and identify Radar emission. Detection is their first issue, since, once a signal is detected, the measurement is carried out by demodulating (in coherent or non coherent way, depending on the receiver) the waveform and the identification is finally performed on library basis.

A. *Next Generation of ESM Receivers*

ESM Receivers, even if in a slower way with respect to Radar Receivers, have followed an evolution process: next generation Receivers combine Super-Heterodyne (SH) architecture with fully digital processing.

In ELT, the technology evolution of Analog-to-Digital Converters (ADC) and Field Programmable Gate Arrays (FPGA) for fast Digital signal Processing (DSP) have allowed the development of a Dual Channel Digital Receiver board (DRx) with more than 1 GS/s of sampling rate and the availability of resources to perform the processing of the whole sampled bandwidth.

The SH DRx based Receiver is shown in Fig. 1. The sampling is performed directly at Intermediate Frequency (IF) level, involving just one ADC (see [1] as example), and is followed by a digital filter bank. The filter bank takes a real (high rate) signal in input and produces in output a number of complex signals (at decimated rate) each one representing the complex envelope relative to the corresponding sub-band; decimation is due to the fact that each sub-band signal, having been filtered, needs a lower sampling rate with respect to the ADC signal; in [2] polyphase techniques are shown for the implementation of uniform filter banks by means of Discrete Fourier Transform (DFT).

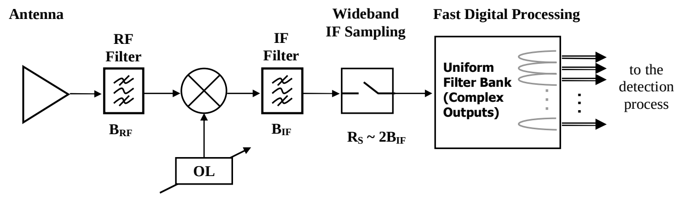

Fig. 1 SH Receiver with DRx

B. *Definition of a LPI Radar signal*

In the present work we assume phase-modulated signals as representative of LPI signals; in any case the results maintain their validity also with frequency modulated signals (Chirp, FMCW) as long as they have a bandwidth comparable with the one of phase-modulated signals.

A radar pulse $s(t)$ is defined in the following way

$$s(t) = A \cdot \cos[2\pi f_0 t + \boldsymbol{\Phi}(t)] \cdot \mathrm{rect}_{PW}(t) \qquad \textbf{(1)}$$

where:

- A is the signal amplitude equal to $\sqrt{2 \cdot E/PW}$, being E the pulse energy and PW the pulse width;
- f_0 is the carrier frequency (it is assumed that $f_0 >> 1/PW$);
- $\boldsymbol{\Phi}(t)$ is a given function carrying the phase information.

The $\mathrm{rect}_{PW}(t)$ function is defined as follows

$$\mathrm{rect}_{PW}(t) = \begin{cases} 1 & 0 \le t \le PW \\ 0 & otherwise \end{cases} \Leftrightarrow$$

In case of unmodulated pulse, e.g. non LPI Radar signal, it is assumed

$$\Phi(t) = \varphi \ \ (constant) \qquad \textbf{(2)}$$

In case of modulated pulse, e.g. LPI Radar signal, it is assumed

$$\Phi(t) = \sum_{k=0}^{Nchip-1} \phi[k] \cdot rect_{Tchip}(t - k \cdot Tchip) \quad (3)$$

In (3) T_{CHIP}, which is equal to PW/N_{CHIP}, is the length of the single sub-pulse. The sequence $\phi[k]$ (to maintain the generic approach), is assumed to be a discrete sequence of random variables: $\phi[k]$ are assumed to be independent, identically distributed (i.i.d.) and uniformly distributed over 2π.

Under previous assumptions, the one-sided energy spectrum for unmodulated pulse is (assuming $f_0 >> 1/PW$)

$$|S(f)|^2 \cong E \cdot PW \cdot sinc[\pi \cdot (f - f_0) \cdot PW]^2 \quad (4)$$

Where $S(f)$ denotes the Fourier Transform of $s(t)$ and sinc[x] = sin(x)/x.

In case of modulated pulse the expected energy spectrum is (under the assumption that $f_0 >> 1/Tchip$)

$$E\left\{|S(f)|^2\right\} \cong E \cdot Tchip \cdot sinc[\pi \cdot (f - f_0) \cdot Tchip]^2 \quad (5)$$

In the above expression the expectation is taken over the random variables $\phi[k]$. The approach used to derive the expected energy spectrum is the same used in Communication Theory (see [3]) to estimate the spectra of Phase Shift Keying (PSK) modulated signals: since the ESM does not have any knowledge of the phase modulating sequence the assumption of a white sequence of random phases (uniformly distributed over 2π) is the most generalized one.

Obviously, by definition of one-sided spectrum, it is found that, in both cases above reported

$$\int_0^{+\infty} |S(f)|^2 df = E \quad (6)$$

The instantaneous power (e.g. half the squared amplitude) in the time domain is equal to E/PW for any (modulated or not) radar pulse having energy E.

The peak level of its energy spectrum (e.g. the energy spectral density) is equal to

$$|S(f_0)|^2 \cong \begin{cases} E \cdot PW & \Leftrightarrow \textit{unmodulated pulses} \\ E \cdot Tchip = E \cdot \dfrac{PW}{RC} & \Leftrightarrow \textit{modulated pulses} \end{cases} \quad (7)$$

In (7) RC = PW/T_{CHIP}, stands for Compression Ratio: this parameter represents the processing gain of the LPI signal (see [4], [5], [6], [7]).

The meaning of RC in frequency domain is also evident: the energy spectrum of a modulated signal, having a given energy E and pulse width PW, shall result (in general) RC times wider and RC times lower than an unmodulated pulse having the same E and PW.

C. Signal and Noise models

Signal and noise models are defined for the SH Receiver with fully digital channelisation processing (shown in Fig. 1) the signal is observed after the Uniform Filter Bank. It is assumed that:

- high rate sampling is carried out at a time step $T_S = 1/R_S$;
- digital multi-rate processing, which implements a Uniform Filter Bank, takes the real wideband input stream (immediately after the ADC) and produces several complex outputs;
- all filters have the same noise equivalent bandwidth B_N so their complex outputs are down-sampled at time step $T_C = 1/B_N$.

At the output of any filter (when signal is present) it is found that

$$r[k] = s[k] + n_C[k] + j \cdot n_S[k] \quad (8)$$

Where $n_C[k]$ and $n_S[k]$ are independent Additive White Gaussian Noise (AWGN) discrete-time processes; the following inequalities hold for samples extracted from those sequences

$$\begin{aligned} &E\{n_C[k] \cdot n_C[h+k]\} = (N_0/2) \cdot \delta[h] \\ &E\{n_S[k] \cdot n_S[h+k]\} = (N_0/2) \cdot \delta[h] \\ &E\{n_C[k] \cdot n_S[h]\} = 0 \quad \forall\ k,h \end{aligned} \quad (9)$$

The signal is defined in the following way

$$s[k] = \sqrt{E \cdot T_C/PW} \cdot \exp[j \cdot 2\pi \cdot f_R \cdot kT_C + j \cdot \Phi(kT_C)] \quad (10)$$

In equation (10) f_R represents a residual frequency with respect to the centre of the filter from which the signal is output; obviously it turns out that $|f_R| < 0.5/T_C$; $\Phi(kT_C)$ is the time sampled sequence $\Phi(t)$; it has to be noted that the amplitude of the signal sample at the generic k^{th} epoch depends upon T_C since the longer the coherent integration time the higher the signal energy obtained.

Concerning T_C we find the following constraints:

- For *unmodulated* pulses: $T_C \le PW$ or $B_N \ge 1/PW$: this translates in practice in a correct filtering of the incoming pulse (since 1/PW is approximately the signal bandwidth).
- For *modulated* pulses: $T_C \le T_{CHIP}$ or $B_N \ge 1/T_{CHIP}$: this means that, due to the lack of knowledge of the modulation, the pulse can be coherently filtered (in general) just for a time not exceeding T_{CHIP}.

The above model is quite ideal since nonzero rise/fall times of radar pulses are ignored as well as some residual Amplitude Modulation (AM) due to the filtering; however it is very useful to understand which is the best achievable SNR, as it shall be seen in the next section.

D. Best SNR achievable by a coherent ESM Receiver

For a coherent ESM Receiver, intended as a receiver producing signals defined in (8), (9) and (10), the SNR can be evaluated in the following way

$$SNR \equiv \frac{|s[k]|^2}{Var(n_C[k]) + Var(n_S[h])} \quad (11)$$

With the definition in (11) we have

$$SNR = \frac{E \cdot T_C / PW}{2 \cdot (N_0/2)} = \frac{E}{N_0 \cdot B_N \cdot PW} \quad \textbf{(12)}$$

Assuming B_N = 1/PW with perfect sampling of the filtered pulse (e.g. assuming that the signal sample has been taken at its peak amplitude) the SNR, for unmodulated pulses, becomes

$$SNR_{BEST\_UNMODULATED} = \frac{E}{N_0} \quad \textbf{(13)}$$

In case of modulated pulses, assuming $B_N = 1/T_{CHIP}$

$$SNR_{BEST\_MODULATED} = \frac{E}{N_0} \cdot \frac{Tchip}{PW} = \frac{1}{RC} \cdot \frac{E}{N_0} \quad \textbf{(14)}$$

The above relationship is very important from a theoretical point of view: even for unmodulated pulses, is possible that all energy can be recovered by an ESM with coherent processing (under the assumption to have filters whose bandwidth matches the pulse bandwidth), for phase modulated LPI pulses this is not possible. The maximum achievable SNR is in general reduced by a factor equal to RC.

II. THE CONCEPT OF CROSS-CORRELATION

In the present section the Receiver block diagram is defined and the relative SNR is found not only for Pulsed signals but also for Continuous Wave (CW) signals.

A. *The System: block diagram and top level considerations*

The full Receiver consists in two equal branches (each one corresponding to a SH with wideband sampling and fully digital channelisation processing); however the processing does not stop here but continues by multiplying the complex output of each filter of the branch 1 with the corresponding complex conjugate output of the branch 2 and accumulating the result. The system block diagram is shown in Fig. 2 (the block referred to as SH Receiver with Wideband sampling and digital Filter Bank is the one depicted in Fig. 1).

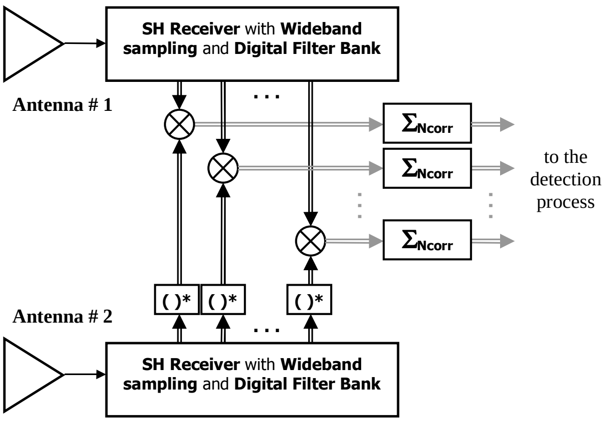

Fig. 2 System Block diagram

It must be noted that the proposed receiver does not evaluate the Cross-Correlation function but only the Cross Energy, e.g. the Cross-Correlation function evaluated at the zero time lag.

The output of the generic Cross-Correlator is defined as

$$XC_{[M]} = \sum_{k=0}^{Ncorr-1} r_{1,M}[k] \cdot r_{2,M}[k]^{*} \quad \textbf{(15)}$$

In equation (15) $r_{1,M}[k]$ and $r_{2,M}[k]$ indicate respectively the outputs of M^{th} filter of branch 1 and M^{th} filter of branch 2 (their signal and noise models have been defined in I.C), N_{CORR} is the number of product samples accumulated (in the following we will refer also to $T_{CORR} = T_C \times N_{CORR}$ as the correlation time).

B. *The output SNR*

The SNR at the output of the Cross-Correlator (which we refer to as SNR_{OUT}), when a signal is present, is defined as

$$SNR_{OUT} \equiv \frac{E\{Re[XC_{[M]}]\}^2 + E\{Im[XC_{[M]}]\}^2}{Var(Re[XC_{[M]}]) + Var(Im[XC_{[M]}])} \quad \textbf{(16)}$$

The above definition is operative since it allows the SNR evaluation not only from a theoretical point of view but also in case of simulation and measure: in fact, once taken several (complex) values of the Cross-Correlator output, a simple statistical analysis provides the SNR estimate.

1) *SNR_{OUT} for Pulsed signals*

The SNR for Pulsed signals is evaluated taking into account that the mean value is nonzero only when the signal is present, whereas noise processes are always present; using (8), (9) and (10) in conjunction with (15) and (16), the following relationship is obtained

$$SNR_{OUT} = \begin{cases} \dfrac{E^2}{N_0^2 \cdot B_N \cdot T_{CORR} + 2 \cdot E \cdot N_0} & T_{CORR} > PW \\[2ex] \dfrac{E^2 \cdot T_{CORR}^2 / PW^2}{N_0^2 \cdot B_N \cdot T_{CORR} + 2 \cdot E \cdot N_0 \cdot T_{CORR}/PW} & T_{CORR} \le PW \end{cases} \quad \textbf{(17)}$$

Using expression (12) as SNR_{IN}, e.g. as the input SNR, the above expression becomes

$$SNR_{OUT} = \begin{cases} \dfrac{B_N \cdot PW \cdot SNR_{IN}^2}{(T_{CORR}/PW) + 2 \cdot SNR_{IN}} & T_{CORR} > PW \\[2ex] \dfrac{B_N \cdot T_{CORR} \cdot SNR_{IN}^2}{1 + 2 \cdot SNR_{IN}} & T_{CORR} \le PW \end{cases} \quad \textbf{(18)}$$

The found expressions agree with those reported in [8] and [9]: for LPI Pulsed signals a value of T_{CORR} exceeding PW results useless since the Cross-Correlation Receiver, once the signal is not more present, integrates the product of noise processes.

2) *SNR_{OUT} for CW signals*

Up to now Pulsed signals have been taken into account, however the case of CW is quite similar: a CW signal $s_{CW}(t)$ is defined in the following way

$$s_{CW}(t) = A \cdot \cos[2\pi f_0 t + \Phi_{CW}(t)] \quad \textbf{(19)}$$

Where $A = \sqrt{2P_{SIG}}$, being P_{SIG} the signal power, and $\Phi_{CW}(t)$ can be defined as

$$\Phi_{CW}(t) = \sum_{h=-\infty}^{+\infty} \Phi(t - h \cdot Nchip \cdot Tchip) \quad \textbf{(20)}$$

being $\Phi(t)$ expressed in (2).

In case of CW signals, the signal power has to be considered, instead of its energy, and the expression of SNR_{IN} becomes

$$SNR_{IN} = \frac{P_{SIG}}{N_0 \cdot B_N} \quad \textbf{(21)}$$

The expression of SNR_{OUT} becomes

$$SNR_{OUT} = \frac{B_N \cdot T_{CORR} \cdot SNR_{IN}^2}{1 + 2 \cdot SNR_{IN}} \Leftrightarrow \textit{CW signal} \quad \textbf{(22)}$$

CW LPI signals allow the use of any value of T_{CORR} in order to increase the output SNR.

III. System Configuration & Test Cases

In the present section main system parameters shall be provided as well as test signals characteristics.

A. System configuration

The system block diagram has been already shown. The following parameters have been considered:

- B_N = 80 MHz → T_C = 12.5 ns.
- N_{CORR} = 800 → T_{CORR} = 10 µs.
- SNR_{IN} (which represents the SNR at the output of the channeliser, e.g. over B_N = 80 MHz) ranging from -20 dB up to 30 dB.

B. Test Signals

Test signals used in both simulations and measurements were LPI CW Signals obtained by replicating a PSK code. CW Signals used are:

- CW Signal obtained by repeating a Barker 13 (implemented with B-PSK modulation) with T_{CHIP} = 100 ns and repetition period equal to 1.3 µs.
- CW Signal obtained by repeating Frank 64 (8-PSK modulation) with T_{CHIP} = 100 ns and repetition period equal to 6.4 µs.

The use of CW Signals has simplified the SNR estimation since the Cross-Correlator has been used in free running mode with the signal always present; the collected data have been elaborated with (12) to estimate SNR_{OUT}. A CW unmodulated Signal (e.g. a pure tone) has also been used as a benchmark.

IV. Results: Theory, Simulation and Measure

Results are shown in Fig. 3: in the same picture the theoretical performance (equation (22) with continuous thick line) is depicted together with simulated performances (continuous thin lines) and measured performances (markers).

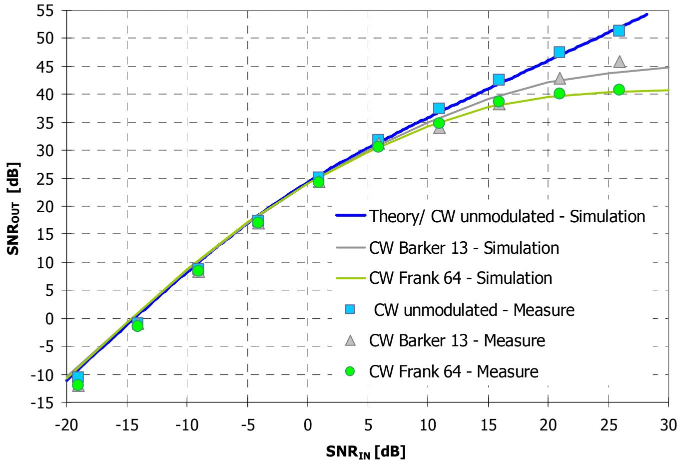

Fig. 3 SNR_{OUT} performances

In case of CW unmodulated signal, theory, simulation and measure are completely aligned with each other. Simulations and measures of CW modulated signals follow theoretical predicted performances up to SNR_{OUT} values less than about 30 dB; then, as SNR_{IN} increases, their actual SNR_{OUT} is less than the predicted one. This effect is due to a residual AM effect induced by the filtering on modulated signals.

Theoretical performances have been anyway able to predict the SNR behaviour of the Cross-Correlation Receiver at low/medium values of SNR_{IN}: which is our real case of interest.

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