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Value of information software: defense applications

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***Abstract* — We exhibit a software developed by us, that allows the representation of decision trees, calculation of the expected value of perfect and imperfect information of a subset of its random nodes in relation to any chosen decision node and attachment of a Bayesian network to any one of its random nodes (with the purpose of relating the random variable that governs it to other ones, thus making easier the assignment of probabilities to its outward edges). An application to a stylized model of deterrence is shown and other ones are suggested.**

***Keywords* — Value of information, deterrence, software**

I. INTRODUCTION

The international market has a certain number of companies that develop and trade software for decision support based upon decision trees and influence diagrams; nevertheless, none of them have routines to calculate the expected value of information in decision trees (some have it for influence diagrams). Mathematical models in the defense industry suffer from high uncertainty in its estimated parameters, given the subjective nature of threats and the inherent fuzziness of socioeconomic systems; as a result, one is always tempted to conduct research to narrow those uncertainties. But how much effort should be put in the quest to pinpoint the model parameters? The answer is found by calculating the value of the information gained (when narrowing the uncertainty) in relation to decisions whose payoffs can be quantified.

The software we present here is an innovation that will contribute to the solution of the problem of assessing how much is it worth investing in research in the defense sector, be it on science and technology be it on information gathering and analysis in general.

II. VALUE OF INFORMATION

"Value of perfect information" (VoPI) is the difference between the expected value of a decision made after solving the uncertainties that affect its results and the decision made replacing the uncertainties by their expected values.

The concept dates back to the 50s of the past century [1] and is already incorporated in textbooks on decision theory [2] and [3].

Formally, the definition is, in the case of risk-neutrality:

$$\text{VoPI} = E[\max f(r,x)] - \max E[f(r,x)] \qquad (1)$$

Where r is a vector of random variables and x is a vector of decision variables. The maximization is always relative to x.

The information is called perfect in this case, because the optimal choice of x in the first term is made knowing exactly which value the random vector r assumed. In most cases, nevertheless, what is achieved is only a partial reduction in the uncertainty of r, which means narrowing its probability distribution with the help of experts; in such cases, what we get is the value of imperfect information (VoII).

References [4] and [5] describe some of the many applications of VoI.

III. PSEUDO-CODE

Although the definition of value of information given above is valid for discrete or continuous variables, the application to decision trees requires that both x and r be discrete.

The software gives two options to the user: calculation via the exact method or approximation via Monte Carlo simulation. The latter is useful, when the former takes too much time (big trees).

Exact calculation

The pseudo-code below gives the recipe for the exact method of calculation of both perfect and imperfect information for a tree with K random nodes:

1. Let $r=[r_1, r_2,\ldots,r_k]$, $k<K$, be the vector of random nodes, whose total value of information one wants.
2. Let $s=[s_1, s_2,\ldots,s_k]$ be the opinions of experts about r
3. Get $P(r)$ and $P(s|r)$
4. Obtain $P(s)=\sum_r P(s|r)P(r)$ and $P(r|s)=P(s|r)P(r)/P(s)$
5. Let $-r$ be the vector of random nodes not present in r
6. Let $f_{-r}(r,x)=E_{-r}(f)$, that is the expected value of the decision node, relative to which the value of information is desired, over the random nodes in $-r$.
7. For each possible realization of the vector s,
Calculate $E_{r|s}[f_{-r}(r,x)]= \sum_r f_{-r}(r,x)\, P(r|s)$
8. Let $g(s)= \max E_{r|s}[f_{-r}(r,x)]$ (maximizations are always on x)
9. Calculate $E_s[g(s)]= \sum_s g(s)P(s)$
10. Let $V_0= \max E[f(r,x)$, that is the value of the subtree whose root is the decision node relative to which the VoI is desired.
11. Get the exact value of imperfect information by
$V_{II}=E_s\{\max E_{r|s}[f_{-r}(r,x)]\}-V_0$
//Observation: if $s=r$, this is the value of perfect information//

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Aproximate calculation via Monte Carlo

1. Get a sample $\{s^1, s^2,\ldots,s^n\}$ *of s according to P(s)as obtained in step 4 of the exact method.*
2. For each s^i *of the sample, calculate*
$h(s^i,x)= \sum_r f_{-r}(r,x)\, P(r|s^i)$
//Observation: easy, because of the hierarchical structure)//
3. Get the answer by
$V_{II}=(1/n) \sum_i \max h(s^i,x) - V_0$

IV. PERFORMANCE OF THE SOFTWARE

In order to test the power of the software, we created the tree of Fig. 1 (a key in the menu bar allows the automatic optimization of the shape of the tree). It has 27 random nodes and 81 utility nodes, the values of which were drawn from a normal random variable with zero mean and standard deviation 50.

The calculation of the VoPI of a subset of 11 adjacent nodes took 1 minute in a desktop computer running windows XP with AMD Athlon II X2 B24 dual core processor 3 GHz and 4 GB RAM. The corresponding hand computation in this case would require the building of an inverted tree (random nodes first) with about 400 million nodes and the calculation of its value, to be later subtracted from that of the original tree, which would be humanly impossible. With 7 randomly selected nodes, it took 1:30 min. This apparent discrepancy is due to the number of decision nodes involved in the calculations; in the former case, only 4 of them; in the latter, 13, the total number of nodes (random or decision) present in the inverted tree being 3^18 in both cases.

Notice that the inverted tree that is needed to calculate the VoPI of the set of all random nodes has a total of 3^27x3^13=3^40~1.2x10^19 nodes, which, at that speed, would take 60 thousand years. In [8], bounds are obtained for VoIP, that mitigate this so called “dimensionality curse” in special cases. The VoPI of the set of all random nodes that descend from node 3 in relation to the central node is 36.1; the same for node 4 is 30.7 and the same for node 5 is 47.7.

Using the Monte Carlo routine of our software, those 60,000 years are reduced to 12 seconds; the result is VoI=69 +/-7 (standard error), when all the random nodes are selected and only 10 samples of the tree are used. Using 100 samples, we get VoI=61 +/-2 in 2 minutes. The corresponding histograms are accepted as Gaussian by standard goodness of fit tests and their standard deviations decrease as expected (proportionally to the square root of the sample size), while the sample average converges as expected, thus showing the consistence of the estimator.

The results were compared with hand calculations in simple trees, no discrepancy being observed. Given the inexistence (to the best of our knowledge) of any other software capable of calculating the value of information in decision trees, comparisons with those were impossible.

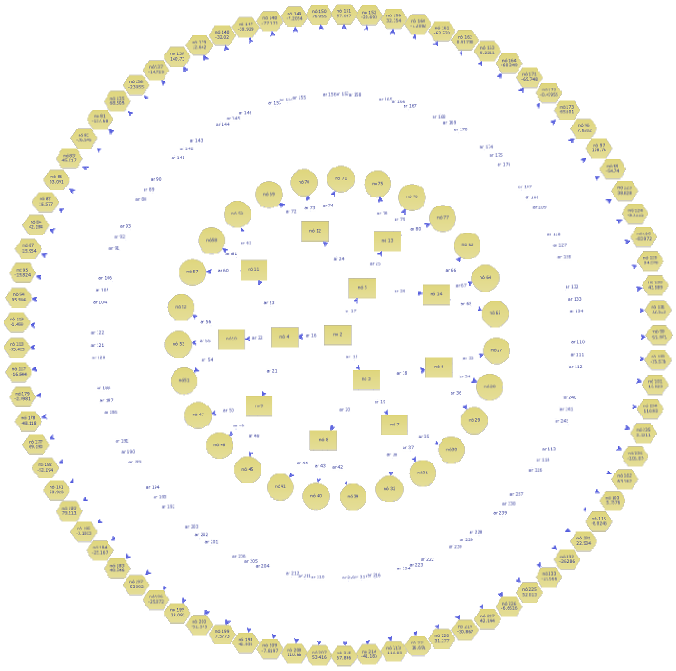

Fig. 1. Test tree.

V. EXAMPLES OF POSSIBLE APPLICATIONS

Some examples can demonstrate the capabilities of our software. First, the simpler non-degenerate case:

Investment

The military is considering the acquisition of a new equipment, but doesn’t know if it will be worth it, because of the uncertainty about future threats serious enough to require their use as well as the effectiveness of the equipment in staving off those threats. The investment would be $1 million. If, in the future, there happen to be threats that demand the use of the improvements, $4 million are spared for the country due to avoidance of damages caused by the enemy; otherwise, the improvements would have served nothing. Suppose the probability of a future serious threat occurring is assessed as 0.5. How much would it be worth investing on the resolution of the uncertainty about future threats before making the decision of acquisition?

A decision tree to represent this example can be seen in Fig. 2. In this example, as the events are equally likely, the probability of the edges could have been omitted when building the tree with the software.

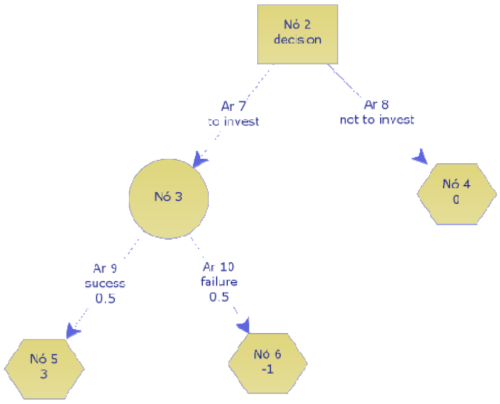

Fig. 2. Decision tree of the investment example (rectangles represent decision nodes; circles are for random ones and hexagons for payoffs; nodes (Nó) and edges (Ar) are automatically numbered).

Value of perfect information (VoPI)

The information is considered perfect when it eliminates all uncertainty about random events. We call VoPI the maximum amount of utils (the payoff unit) that would be worth paying to acquire perfect information about a subset of random nodes.

After creating the decision tree of Fig. 2, choose "Analyze" in the top menu and then "VoPI". A new window appears to set VoPI parameters. Check "nó 3" and press "Calculate VoPI". The result will appear: 0.5.

This result means that it is worth investing up to $0.5 million, to know for sure if there will be threats in the future that require the use of the new equipment and facilities.

In the general case, one can choose a subset of random nodes and a single decision node. The result will answer the question: "How much is it worth investing to eliminate the uncertainties related to the set of random nodes before making the decision indicated by the decision node?".

Value of imperfect information (VoII)

In the case of VoII, the information obtained doesn't completely eliminate the uncertainty. Consider that the C4SIR (Command, Control, Communications, Computers, Intelligence, Surveillance and Reconnaissance) division in charge of reducing the uncertainty about future threats has the following hit rate:

- In cases in which those serious threats will happen, the lab guesses correctly 90% of the time;
- In cases where the threats won't happen, it guesses correctly 60% of the time.

In order to obtain the VoII, after creating the decision tree, choose "Analyze" in the top menu and then "VoII." A new window appears to set the VoII parameters. Check "nó 3" and press "Define conditional prob."; another window will pop up.

Four fields are required. To fill them, it should be noted that, in Figure 2, the edge 9 represents success and the edge 10 represents failure. So, the filling should be done as follows:

- P(9*/9) is the probability of the division forecasting threats when the threat will happen. Therefore, it should be filled with 0.9;
- P(9*/10) is the probability of the division forecasting threats when the threat won't happen. Therefore, it should be filled with 0.4 (1-0.6);
- P(10*/9) is the probability of the division forecasting no threat when threats will happen. Therefore, it should be filled with 0.1 (1-0.9);
- P(10*/10) is the probability of the division forecasting no threat when threats won't happen. Therefore, it should be filled with 0.6;

Note that the sum of each column must be 1 (version 1.1 of the software automatically fills the remaining probability to sum 1).

After filling all the fields, press "Ok". The window with conditional probabilities will close and the button to calculate the VoII will be enabled. Press "Calculate VoII" and the result will be displayed: 0.1500001. This means that it is worth investing up to $0.15 million in the C4SI division, to get the threats assessment, considering its credibility.

Deterrence

Consider a modification of Powell's model [6]. Two rival countries are disputing a certain resource and try to bargain it, while keeping open the possibility of war. One part, S, is satisfied with the *status quo*, but the other one, D, is not. The dissatisfied part will hit the satisfied one, if it thinks the cost of the physical confrontation is less than the expected value of the chunk it will get after the war ends.

Suppose that the total amount of the disputed resource is 1 and the potential raider D (dissatisfied) already has a quantity q of it, while S (the satisfied part) has b-q . Let R = (r,1-r) be the Nash equilibrium partition for (D,S), obtained by adding half of the surplus (1-b) (the synergy gains of an agreement) to each side. Then D will not attack if p-r < d, where d is the cost of the war for it and p is the probability of it winning. Once the confrontation occurs, the winner takes all the resources; thus the expected result of war is (p-d,1-p-s), where s is the cost of war for S. If D refrains from attacking, nothing changes, the result being (q,b-q).

Now, keeping p<1 requires a certain level of capacity for the armed forces of S, which has a cost; suppose this cost is K/p-K (=K(1-p)/p). Also suppose that the constitution of country S forbids wars of conquest, so that, even if the expected result of war for it is favorable, it refrains from attacking. Then the goal of S is to lower p just enough to avoid war with D, that is so that p < r+d. But S only has a crude estimate of d, namely, it only knows that d is equally likely to assume any one of the values d_1, d_2 or d_3. Also suppose that there are only three possible capacity levels for the army of S, corresponding to three different values of p: p_1, p_2 and p_3.

How much is it worthwhile investing in intelligence, in order to know for sure which one is the true value of d?

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Suppose that b=0.9, q=0.2, s=0.2, K=0.1, d_1=0.1, d_2=0.2, d_3=0.3, p_1=0.3, p_2=0.4 and p_3= 0.5.

Then the Nash bargaining equilibrium has r=0.25 (surplus 1-b=0.1; half that, 0.05, going to each side). The costs of keeping the army capacity levels corresponding to p_1, p_2 and p_3 are, respectively: 0.33, 0.25 and 0.2. Table 1 shows the results and the corresponding payoffs of S for each one of the nine possible combinations of p and d (reminding that peace is guaranteed whenever p<r+d, otherwise it is war); the payoffs are all negative and made of the cost of maintaining the army plus (in case of war) the postwar loss of S in relation to the Nash bargain equilibrium (in the second column, 0.75-0.60; in the third one, 0.75-0.50) and the destruction cost of war s. Fig. 3 shows the corresponding decision tree.

Table 1: Possible results of the deterrence game.

	p_1=0.3	p_2=0.4	p_3= 0.5
d_1=0.1	Peace -0.23	War -0.50	War -0.55
d_2=0.2	Peace -0.23	Peace -0.15	War -0.55
d_3=0.3	Peace -0.23	Peace -0.15	Peace -0.10

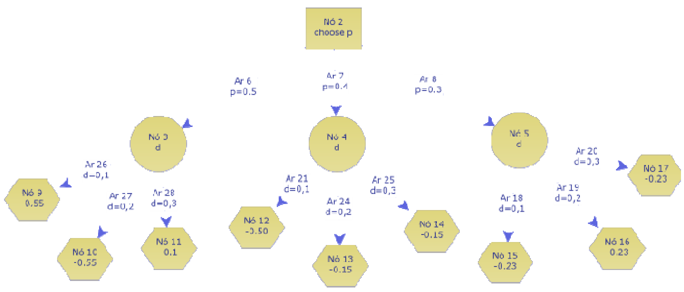

Fig. 3. Decision tree of the deterrence game.

The expected value of perfect information about d, as calculated by our software, is 0.07 (this case is simple enough to allow hand calculation; the value of the original tree is -0.23, while that of the inverted tree is -0.16). As the value of the asset in dispute was normalized to 1, this means that, if it was US$100 billion, it would be worth investing up to US$7 billion in intelligence, to know the exact cost of destruction that D expects to suffer if there is war.

Note that the random nodes 3, 4 and 5 represent the same random variable d. To add this information to the software, go to the top menu "Analyze" and then "dependence of events". Choose fields "Node 3" and "Node 4" and click "Add". Then, a list of edge identifiers will appear, so that the user can select which edge of node 3 is equivalent to which edge of node 4. Make the correct matching.

Press the "Save" button. The equivalence between the nodes 3 and 4 was added to the software and now it can be seen on the top bar.

Click "New equivalence" and repeat the procedure with nodes 3 and 5. Now the program understands that the three nodes represent the same random variable.

War or diplomacy?

We included some routines of the UNBayes package [7] in our software as an aid to determine the probabilities in the tree.

Consider the example "war or diplomacy":

Red country invades part of blue country. Blue has to decide between seeking diplomatic agreement or war. The latter is riskier: if red is strong, the situation of blue can worsen; but, on the other side, if red is weak, blue can get back its territory plus war reparations. In the diplomacy option, the worst case is the permanence of the *status quo*, if red is strong; otherwise, a small gain, if red is weak and, thus, prone to a middle ground agreement. With the payoffs representing utilities of the 4 possible results, this case can be represented by the tree in Fig. 4.

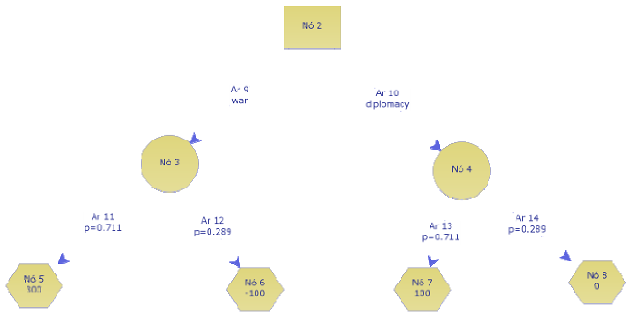

Fig. 4. Decision tree of "war or diplomacy" example.

Now, before making a decision, blue needs to know the probabilities of red being strong or weak. It has two means for that: visual or communications surveillance of red.

- The blue country believes, *a priori*, that visual surveillance analysts will probably (with p=0.7) conclude that red is weak and, thus, with p=0.3, that it is strong.
- Likewise, blue believes, *a priori*, that communications surveillance analysts will probably (with p=0.8) conclude that red is weak and, thus, with p=0.2, that it is strong.
- When both information channels say that red is weak, there is a 90% chance that it is indeed.
- When both channels say red is strong, there is a 95% chance that it is indeed.
- When the photography analysts say red is weak and the radio listening personnel say it is strong, there is a 60% chance of red being weak.
- When the photography analysts say red is strong and the radio listening personnel say it is weak, there is a 50% chance of red being weak.

In order to get P(strong), after creating the tree of Fig. 4, click with the right button at node 3 and select "Associate a Bayesian network." A window for creating Bayesian networks opens. Select "File -> New -> BN" in the top menu. Create a Bayesian Network. Assuming the two sources are statistically independent from each other, the result is P(strong)=0.711 and, thus, P(weak)=0.289. The software automatically modifies the former equiprobabilities to these new ones. As a result, in this example, the blue country should either declare war or pay up to 28.9 utils to know for sure if red is strong or weak, for this is the VoPI of the two random nodes.

Note: in this example, it is important to remember to add the equivalence between nodes 3 and 4, since they represent the same random variable.

Sample screen

Figure 5 exhibits a sample screenshot of the main window illustrating another feature of the software: the subtree to the right was automatically attached to one of the nodes of the original tree. This means that a big decision tree can be built in a decentralized way: each branch of an organization builds its subtree to be attached to the global tree.

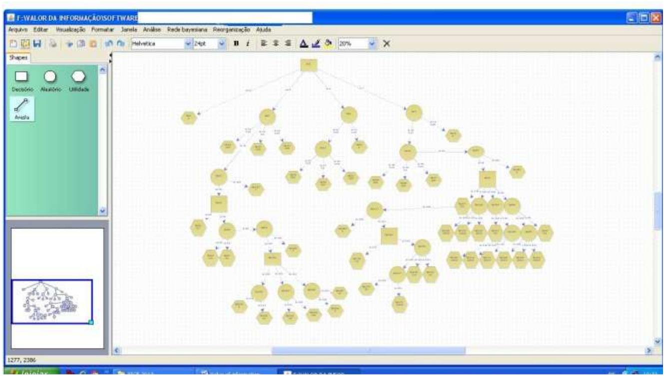

Fig. 5. Sample screen.

V. CONCLUSIONS

With the software here introduced, many tasks involving pricing, resource allocation and risk assessment in the armed forces can be rationally tackled. The calculation of the value of information in big decision trees becomes now feasible, which opens a whole new range of possibilities of analysis not only in the defense sector, but also in the civilian one. It is a dual technological improvement that was lacking in the international market. Among other possibilities, we envision the following:

1. Graphical representation of scenarios that can demand the use of the defense and civil sectors. For this, we are implementing routines to handle big trees.
2. Valuation of big projects in the defense and civil sectors, using the concept of real options.
3. Risk analysis of military operations, using the Bayesian networks module.
4. Prioritization and allocation of resources for research and development in the air force.

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