GNSS Signal Acquisition Below Shannon-Nyquist: A Perspective on Sparse Recovery

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***Abstract*— Acquisition is a crucial process in GNSS systems as it determines which satellites are visible, a course estimation of the Doppler shift and an initial estimation of the time delay between the transmitter and the receiver. Commonly, acquisition is performed by serial search or by circular correlation search. Both processes, although very understood and deployed in practical systems are very time and computational consuming. In this work it is shown that acquisition process is intimately related to the compressive sensing framework, enabling under-sampling of the received signal yet keeping recovery guarantees. Simulation results presented points towards a new and interesting direction in which the GNSS acquisition can be performed using sampling rates far below the Shannon-Nyquist limits, leading to simplification of hardware and software.**

***Keywords*— GNSS, Compressive Sensing, Sparse Optimization, Acquisition**

I. Introduction

The GNSS (Global Navigation Satellite System) is a generic term for the several satellite navigation systems. That system allows small antennas to receive the signal sent continuously by multiples satellites with the goal of determining the receiver position, velocity, and journey time in the Earth [1]. There are several operational systems today, being the two most known the GPS (Global Positioning System), developed by the North American government, and the Russian GLONASS (*GLObal'naya NAvigatsionnaya Sputnikovaya Sistema*). Additionally, there is the European Galileo and the two regionally accessed: the Chinese Beidou and the Indian IRNSS (Indian Regional Navigation Satellite System). Although this work uses the signals models from the GPS to perform the acquisition algorithms presented, the same methodology can be easily adapted to work with the signals models of the others systems.

The GPS system has three segments: 1) the space segment, which consists of a nominal constellation of 31 satellites, distributed in six orbital planes; 2) the control segment, composed of a ground station network responsible for monitoring the satellite transmission, analyzing the receiver perform, and sending commands and data to the constellation; and 3) the user segment, that is mainly composed of receivers, processors, and antennas which receive and treat the L-band signals sent by the GPS satellite constellation [2].

The purpose of a receiver is to determine its own position (latitude, longitude e altitude) at a given moment, based on GNSS signals. To achieve this, a sequence of operations must be done before that the GNSS receiver could propose a navigation solution. Hence, the first receiver task is to identify the visible satellites, which is performed by the GPS signal acquisition process [3].

The GNSS signal acquisition process consists in a search in time (code phase), a search in frequency (Doppler shift) and a search in PRN (Pseudorandom Noise) codes (satellite identification). This process has three main objectives, namely to detect the presence of visible satellites, to determine the delay of the PRN code and to estimate the Doppler shift. There is a large amount of literature on PRN code acquisition, the most known algorithms are detectors in the time domain, but also there are the frequency domain search techniques [4]. In this paper we discuss a comparison between the serial search and circular correlation search algorithms.

Acquisition algorithms have been studied for several years and some approaches as the serial search and circular correlation search have attracted much attention in order to lead to efficient and quick solutions. Although consolidated, those algorithms are known as time and computational demanding. Wherefore, in this work, we also present a new possibility to deal with the acquisition process, aiming to reduce its computational complexity and, so, enabling the adoption of simpler hardware and software structures in its implementation. This potential new approach is called sparse acquisition process.

The paper is organized as follows: in section II, the basics of GPS signal is introduced; Section III brings the mathematical concepts in the acquisition process, recalling the traditional approaches to that, and introducing a new possibility in acquisition based on the compressive sensing theory; Section IV deals with the computational complexity of the acquisitions schemes; in Section V, some simulation results are presented, paving the way to the new approach adoption; finally, Section VI brings conclusions and future work possibilities.

II. GPS Signal

The signals generated by the GPS satellite constellation are transmitted at two L-band carrier waves, called L1 and L2, which are generated from a base frequency $f_0 = 10.23\,\text{MHz}$ [5]. Mainly, the signal transmitted by a GPS satellite can be divided in three components: 1) Navigation data, which are binary messages containing the Ephemerides, used to calculate the position of the satellite in orbit, and the Almanac, which contains information about all constellation; 2) PRNs, which are random sequences of unique codes for each satellite, used

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to spread the navigation data spectrum; and 3) the Carrier Frequency[4].

There are two types of GPS signal based on the type of PRN used. The C/A (Coarse/Acquisition code) is available to the general public and is used in the commercial GPS receivers since the PRN of each satellite is known. On the other hand, the $P(Y)$ (Precision code) is reserved for military use and the PRNs are stealthy. Therefore, in this work, we used the C/A code to perform the signal acquisition.

Each GPS satellite broadcasts its navigation data at a rate of 50 bps (bits per second). These data are spread with a pseudo-random code that has 1023 chips of length, running at 1.023 MHz (called chipping rate). This means that at every 20 ms a data bit is spread by 20 packets of 1023 C/A code chips [6]. This technique, called Spread Spectrum, is used to increase the bandwidth, allowing the navigation data to be recovered below the noise floor at the receiver by application of the same PRN [4]. Additionally, this allows the GPS satellites to share the same carrier since each one broadcast the information spread by a specific PRN code.

III. GPS Signal Acquisition

The purpose of the GPS signal acquisition process is determining which satellites are visible at a given moment (τ) and provide a coarse estimation of the frequency and PRN code chip shift at the receiver. Frequency shifts are caused by Doppler effect and by imprecisions in the receiver local oscillator. PRN code chip shift is caused by the time difference between the instant when the code is generated at the satellite and the instant when it is received.

The acquisition process is done by correlating the received signal (s) with a PRN signal (c). The correlation process is a measure of similarity between signals and can be classified into two types: cross-correlation, the measurement of similarity between distinct signals, and auto-correlation, the measurement of similarity between a received signal and its replica. In this work, the signal used as the replica is the C/A PRN code, since it is the code available for civil purposes. It's important to say that the correlation requires that the chips of the C/A codes, which is a binary sequences of 0 and 1, are transformed into code states (+1 e -1) [1].

There are currently 31 operational satellites in the GPS constellation [2], and each one has its own PRN sequence. Therefore, the acquisition process correlates the received signal with each one of the 31 available codes. However, only the correlation is not sufficient to find the visible satellites since there are many sources of errors that can produce inaccuracies, as the receiver noise, interference, multipath and the ionospheric refraction [7]. Also there is the influence of the Doppler effect and the time shift. The Doppler effect occurs due to the motion of the satellites, the motion of the receivers, and the Earth rotation. The typical Doppler interval, when the receiver is at rest, is ± 5 kHz. When the receiver is in motion, the deviation due to this motion must be added to the deviation caused by the motion of the satellites and the Earth rotation. In such case, the maximum interval is ± 10 kHz [6]. On the other hand, the time shift is caused by the delay of the signal propagation between the satellites and the receiver. This effect may cause changes in the structure, size, and rate of the signal [4].

Therefore, in the acquisition process is necessary to look for a correlation magnitude that shows the code chip shift and frequency shift. That means that the algorithm must search for the Doppler shift in a search window of 20 kHz and for the code delay in $1023.f_s/f_{C/A}$ possibilities, where f_s is the frequency sampling at the receiver and $f_{C/A} = 1023\,\text{MHz}$ is the frequency of the code chip. This search is necessary because the GPS signal is spread spectrally and can only be detected if it is unassembled by its replica, which has to be precisely aligned with the received signal.

Beyond that, in order to handle the GPS signal by software, it is usually indicated sampling the signal following the Shannon-Nyquist's limits[8]. According to this theorem, the sampling frequency f_s must be at least twice the frequency of the C/A code. However as it is desired a higher accuracy, the sampling frequency considered in this work is $f_s = 8.192\,\text{MHz}$.

Thus, considering that the frequency shift range is 20 MHz and that the search algorithm uses a step size of 500 Hz called bin, for each one of those bins will be performed a correlation between the received signal (already sampled by f_s) and the C/A code of each satellite. In such a manner that, for each satellite, those correlations will form a search matrix for the GPS signal sized by 8192×41 cells, as shown in Fig. 1, in which the search cell with the highest peak value indicates the frequency offset and the signal delay.

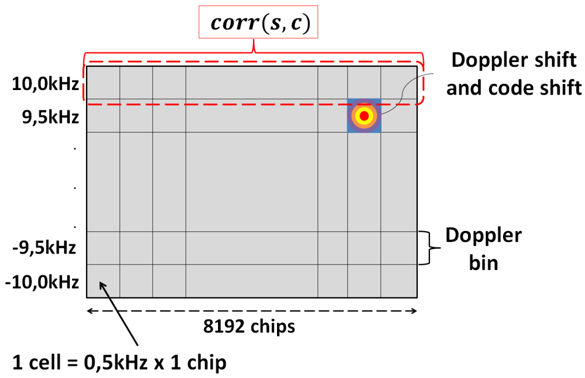

Fig. 1 - Two-dimensional matrix of search acquisition

Since the GPS signal was sampled, the algorithm to search the Doppler and time shift will be performed as in Fig. 2. The input signal s is correlated with a replica of the C/A code sampled at frequency f_s. The methodologies of signal acquisition presented here will differ only in the block called matched filter. The correlation will occur for each frequency bin and for each satellite until finding which satellites are visible. In the following subsections, two classical search algorithms are presented, one in the time domain and the second in the frequency domain. Additionally, an alternative acquisition methodology, based in sparse processing, is also presented.

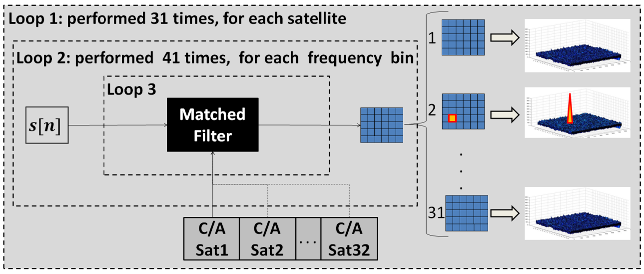

Fig. 2 - GPS signal acquisition general algorithm

A. Serial Search Acquisition Process

The serial search is the simplest algorithm used to GPS signal acquisition. This algorithm is based on non-coherent correlations, and consist of multiplications and sums between the received signal and the known PRN codes. Considering that the received signal is s and the C/A code replica is called c_i, where i indicates the i^{th} satellite, the product between s and c_i is multiplied by the apparent frequency ($f_{(C/A)}$ plus the Doppler shift), generating a signal in phase (I) and in quadrature (Q). The signals I and Q are integrated over every 1 ms and accumulated for N periods to finally be squared summed [5]. The Fig. 3 shows the block diagram of the GPS acquisition serial search algorithm, where j is the index of a new block of data, L is the length of the vectors and k is the quantities of correlation that will be performed for a same bin.

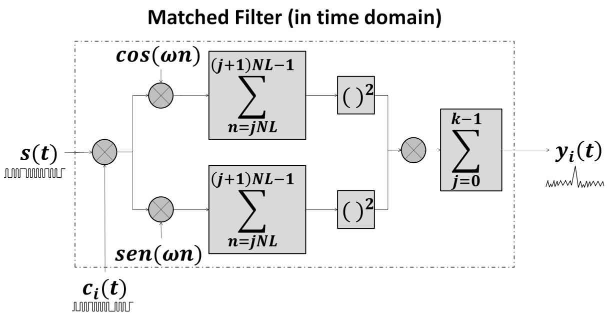

Fig. 3 - Non-coherent correlation in time domain

The output of the matched filter is the result of the correlation between the received signal and the C/A codes of each satellite. If the maximum value found in the search matrix is the higher off all search matrix, the respective satellite is considered to be visible. The correlation for GPS signal acquisition can be represented by [9]:

$$R^2[m] = \sum_{j=0}^{K-1} \left(\left[\sum_{n=jNL}^{(j+1)NL-1} s[n].C[n].\cos(\omega n) \right]^2 + \left[\sum_{n=jNL}^{(j+1)NL-1} s[n].C[n].\sin(\omega n) \right]^2 \right) \quad (1)$$

B. Circular Correlation Acquisition Process

Unlike the serial search, which seeks for a correlation peak sequentially in two dimensions, combining phase and frequency [10], to execute the correlation in the frequency domain the receiver stores a complete data sequence in order to translate it into the frequency domain via the Fourier transform. The vector corresponding to the C/A code used as replica must has the same length of received sequence. The replica is multiplied by the exponential $e^{\omega t}$ in order to seek for the correlation peak in a determined frequency bin, and is also converted to the frequency domain. With both signals in the frequency domain, the correlation is achieved by multiplication of the sequences followed by the inverse of the Fourier transform. This scheme is known as circular correlation through FFT (Fast Fourier Transform) and is given by

$$R[m] = abs(\mathbb{F}^{-1}(\mathbb{F}(\vec{s}[n]).\mathbb{F}(C_i[N^*]))), \quad (2)$$

where $\mathbb{F}$ corresponds to a discrete Fourier transform, $\mathbb{F}^{-1}$ is the inverse discrete Fourier transform, * corresponds to the complex conjugation, and $abs(.)$ is the operator that extracts the absolute value.

In the Fig. 4 is shown the correlation scheme in the frequency domain.

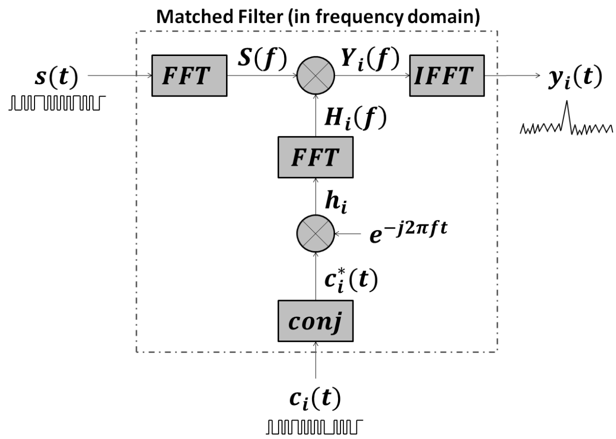

Fig. 4 - Circular correlation in frequency domain

C. Sparse Acquisition Process

As shown in (2), the correlation in frequency domain for a determined frequency bin can be performed as a product of $\mathbb{F}(\vec{s}[n])$ and $\mathbb{F}(C_i[n^*])$, where $n \in 1, ..., N$. The circular correlation can be represented in matrix form as:

$$\theta_i^{(k)} = \mathbf{F}^H(\mathbf{C}\mathbf{F}\vec{s}_k) \quad (3)$$

where $\mathbf{C}$ is a matrix composed by zeros and whose main diagonal is formed by the elements of $\mathbb{F}(C_i[N^*])$, $\vec{s}_k$ is a vector composed with samples of $\vec{s}$ concerning the frame k, an $\mathbf{F}$ is a matrix $N \times N$ corresponding to the DFT operator.

If the frequency bin considered in (3) is approximately correspondent to the Doppler shift and the code used is present in the visible satellites, then $\theta_i^{(k)}$ will present only one large amplitude tap and several low amplitude taps. In other words, $\theta_i^{(k)}$ can be considered sparse and original signal can be written as

$$\vec{s}_k = \mathbf{A}\theta_i^{(k)} + r \tag{4}$$

where r is a Gaussian white noise vector and the system matrix $\mathbf{A}$ is done by

$$\mathbf{A} = \mathbf{F}^H(\mathbf{C}^{-1}\mathbf{F}). \tag{5}$$

The matrix $\mathbf{A}$ is a $N \times N$ square matrix whose columns are formed by the vector $\vec{C}_i$ rotated of m samples, $m \in 0, 1, ..., N-1$, the rotation corresponding to the m^{th} column of $\mathbf{A}$. Thus, each row of the matrix $\mathbf{A}$ is the vector $\vec{C}_i$ with one position offset relative to the previous one row. This configuration indicates that the scalar product between any two different columns of $\mathbf{A}$ produces a result close to zero. In other words, $\mathbf{A}$ can be considered to present low mutual coherence and this fact, allied to $\theta_i^{(k)}$ be highly sparse, allows the introduction of the concepts of compressive sensing for the determination of the chip offset between the received data and the C/A code [11]. Thereby the non-zero term of $\theta_i^{(k)}$ indicates the chip offset, and $\theta_i^{(k)}$ can be obtained:

$$\vec{\theta}_i^{(k)} = \max\ \mathbf{A}^H . \sum_{k=0}^{K-1} \vec{s}_k. \tag{6}$$

Although (6) was exactly the serial search, it can be performed using sub-sampling, based on the compressive sensing framework. This is an indication that the problem can be solved from undersampled sequences and using simpler techniques as the well-known *matching pursuit* (MP) [12] or *orthogonal matching pursuit* (OMP) [13] algorithms.

IV. Computational Complexity

The sample frequency considered in this work is 8.192 MHz, resulting in 8192 samples at each 1 ms. In addition, the frequency range for searching for the Doppler was considered to be ±20 kHz, divided in steps of 500 Hz. This results in a search matrix of size 41×8192. As showed in the Fig. 2, the algorithm is a sequence of nested **FOR** loops, where the first one indicates the search for the satellites, repeated 31 times. The second loop is related to frequency search, executed 41 times, and the third one is related to the displacement of the chips codes.

In the serial acquisition approach, considering that a whole navigation data bit is used in the process, the number of samples per acquisition process will be $N = 1 \times 20 \times 1023 \times 8192 = 167608320$. In this case, the computational complexity is $Op = N \times 8192 \times 41$ real multiplications and the same number of real additions, 8192 for each row in the matrix of search and 41 for each column in the matrix of search. In the end, the serial search algorithm, for all 31 satellite, will have executed $3 \times Op \times 31$ complex operations (multiplications and additions). In conclusion, this algorithm is quite exhaustive and timing consuming.

Considering the same scenario for acquisition in the frequency domain, it will be acquired 20 packets of 8192 samples of the received signal. If the FFT is performed using the Split-Radix algorithm [14], we would have the following tasks for 1 packet (i. e., $n = 8192$):

- The C/A code must be complex conjugated and multiplied by $e^{i\omega t}$. In this case 8192 multiplications should be made, which would give us 8192 real multiplications per cosine and 8192 real multiplications per sine, or $(n+n)$;
- Perform the complex Fourier transform, using the Split-Radix algorithm for 8192 samples of $\vec{s}$. The complex Split-Radix algorithm has complexity $M_c = 2^n(n-3)+4$ for real multiplications, where m is given by $2^m = n$, in our case $m = 13$. The complexity for real additions of this algorithm is $A_c = 3.2n(n-1)+4$ [15].
- Perform a real Fourier transform for the C/A code, using the Split-Radix. The real Split-Radix algorithm has complexity $M_r = 2^{(m-1)}(m-3)+2$ for real multiplications and $A_r = (3n-5) \times 2^{(n-1)}$ for real additions [15].
- After all those operations, we must to apply the inverse complex Fourier transform, also using the Split-Radix algorithm.

Thus, for all satellite, using the algorithm in frequency domain will be executed $31 \times 41 \times 20 \times (2M_c + 2A_c + M_r + A_r + n + n)$ operations. For comparison, the serial algorithm have to performed approximately 164 Tera operations of multiplication and addition for processing 1 data navigation bit. Whereas using the correlation in the frequency domain this number drops to 0.8 Giga operations of multiplications and additions to process the same amount of data.

Now, using the sparse approach, the sample frequency will be the same as $f_C A$, the evaluation of (6) demands N^2 multiplications and $N(N-1) + KN$ sums. In the following section is presented a comparison between the results of the acquisitions schemes for one satellite.

V. Simulation Results

This section presents the results of classical acquisition algorithms (serial search and circular correlation search) and the new proposed methodology, based on sparse processing. In the classical approach, the sampling frequency obeys the Shannon-Nyquist criterion and in this work the sampling frequency used is $f_s = 8MHz$. However, in the sparse processing approach, the system is undersampled, allowing utilization of fewer computational resources, in this case, the sampling frequency used is equal to the C/A code frequency, i. e. $f_s = f_{(C/A)} = 1.023MHz$. All algorithms were developed in Matlab and were also implemented in $C\#$. For the sake of clarity, all the simulations consider navigation data from satellite 1.

The first tests of algorithms are shows in Fig. 5 that presents the result of the serial search process acquisition, Fig. 6 that presents the acquisition of the circular correlation process, and Fig. 7, that shows the result of a acquisition using the sparse approach. The first two results uses the sampling frequency $f_s = 8MHz$ and the third one uses the sampling frequency $f_s = f_{(C/A)} = 1.023MHz$ and considers a sub-sampling by factor 2. For all process the Doppler shift was kept in $0Hz$ and the result of algorithms found the same results for Doppler and code shift, although magnitude peak is higher in the serial search, while the resultant noise, in this case, is lower. Even though the sparse approach be quite noisy it also found the Doppler and code shift.

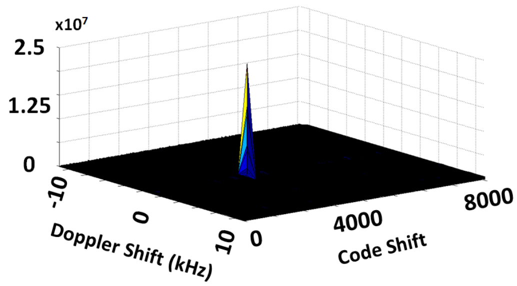

Fig. 5 - Auto-correlation results using serial search.

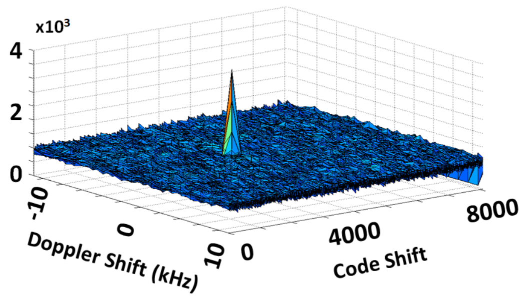

Fig. 6 - Auto-correlation results using circular correlation search.

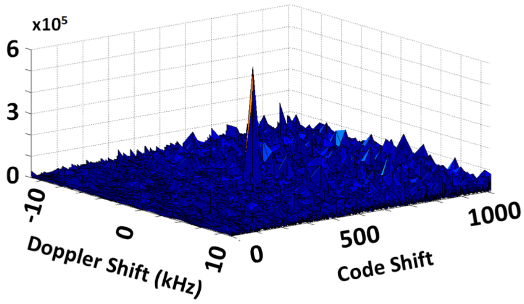

Fig. 7 - Auto-correlation results using the sparse approach.

In the next set of simulations the serial search and circular correlation search were performed considering the sampling frequency f_s below to the Shannon-Nyquist criteria and equal to $f_{C/A}$. The Doppler shift used to the tests was varied from 0 Hz to 200 Hz and the samples equivalent to one whole navigation data bit were used in for the correlation peak determination. A total of 100 Monte-Carlo runs were employed to the mean error calculation. Finally, for the sparse acquisition evaluation were considered sampling frequencies of $f_{C/A}$ divided by 2, 4, 8, and 16. In Fig.8 is shown the evaluation of errors in terms of code shift identification for each one of algorithms considering the increase of the Doppler shift. As expected, the higher the frequency division factor, faster the accumulation of error in the identification of the code shift.

In the Fig. 9, the first result is the serial acquisition with sample frequency of 1.023 MHz and the second is the acquisition in frequency domain with the same sampling frequency.

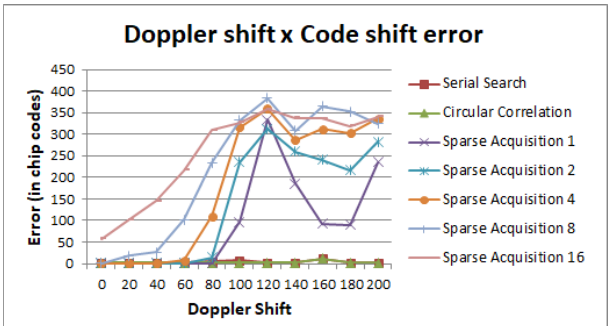

Fig. 8 - Graphics of code shift errors

The third one was the sparse acquisition approach results, considering sub-sampling by factor 2. The next results also uses the sparse approach, but undersampling the received signal by factors four, eight and sixteen positions in the vector of the received signal. It is important to note that even in the undersampling scenarios, the sparse approach brings low error for Doppler as high as 80 Hz. This is an indication that the sparse approach can be used if the frequency bins comprise thinner bandwidths.

Besides, the execution time for the acquisition algorithms for all 31 satellites, considering a sampling frequency equal to the C/A frequency, i. e. $f_s = f_{C/A} = 1.023MHz$, is compered and shown in Fig. 10. The serial search algorithm, although giving a better result, took approximately 78 seconds to perform the acquisition, the acquisition in frequency domain took about 14 seconds. The sparse approach took 12 seconds, 5 seconds, 3 seconds, 1.5 seconds and 0.66 second to perform sparse acquisition jumping one, two, four, eight and sixteen positions in the vector of the received signal. These results show that the sparse processing has potential in save computational resources.

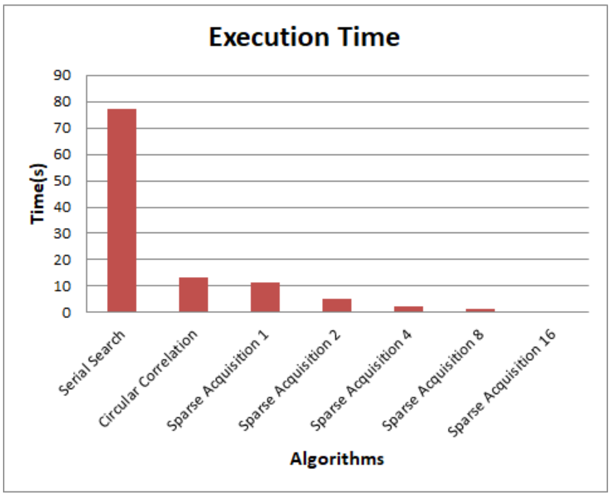

Fig. 10 - Comparison of execution time between all algorithms

VI. Conclusions and Future Work

In this work, a recall in acquisition algorithms for GNSS was performed, considering the classical techniques based in serial search and circular correlation search. These techniques

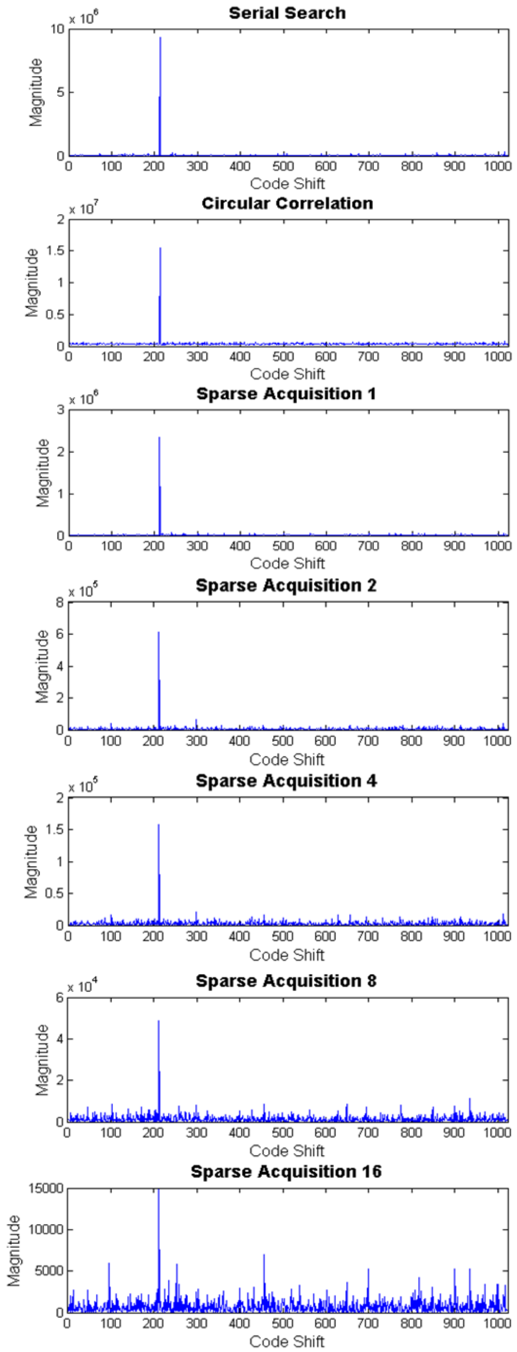

Fig. 9 - Doppler shift 0Hz

were compared in terms of performance and computational performance. The literature claims that the acquisition process based on circular correlation search is faster and computational less intensive than the serial search approach. These claims are validated by the simulation results presented in this work.

On the other hand, a new approach for acquisition, based on the concepts of compressive sensing, is presented. Via algebraic manipulations, it is possible to state that the GNSS acquisition problem can be stated as a sparse recovery problem, benefiting by all the advantages assured by the sparse optimization. This assumption can pave the way to development of a new, faster and less intensive computing acquisition algorithms. Simulation results presented in this work attest this characteristic and highlight the potential of the new technique.

As an orientation to future works, the compressive sensing applied to the GNSS acquisition problem can be mathematically stated and simulations considering more realistic scenarios should be performed. The use of specific algorithms for Doppler and chip shifts joint-optimization is an area that can lead to an important harvest of new and more efficient methodologies in GNSS acquisition procedures.

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